

CHAPTER 6

MAGNETIC FIELD

6.1 Motion of charged particles in a uniform magnetic field:

A particle with positive charge q is moving with velocity v perpendicular to the direction of a uniform magnetic field B as shown in Fig. (6.1).

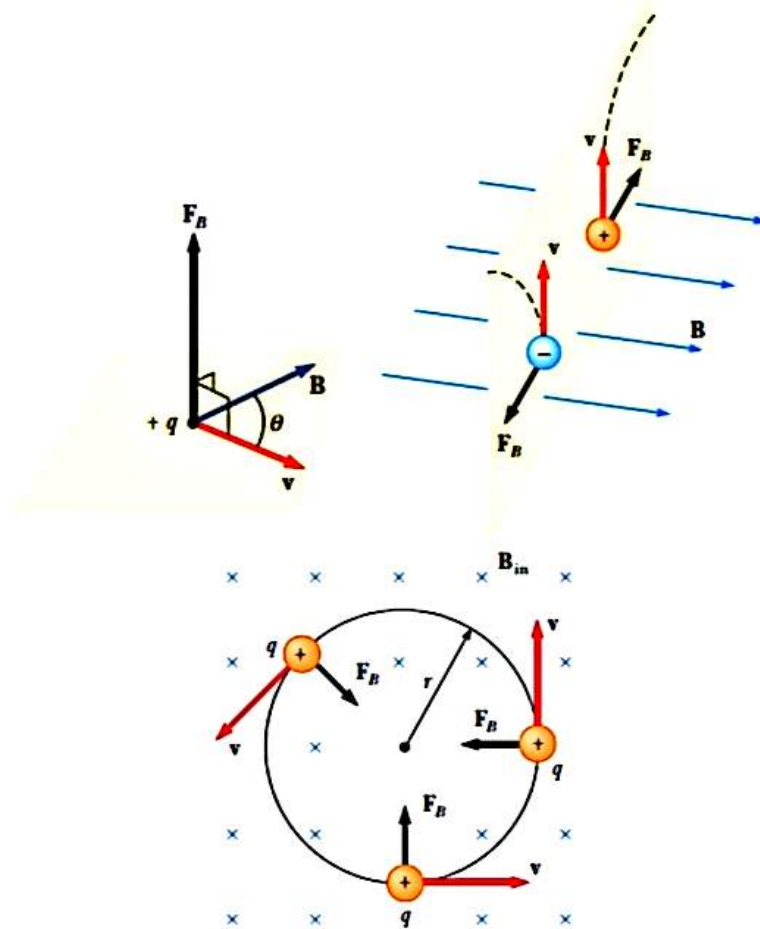


Fig. (6.1)

The magnetic force $F = qvB$ acts on the particles at this point.

If the particle enters the magnetic field by angle Θ , the normal component of the velocity is $v \cos \Theta$ and the acting force will be

$$F = qvB \cos \Theta$$

The force is always perpendicular to the velocity v so it can not change the magnitude of the velocity but only its direction. Thus the magnitude of both F and v are constant. The particle is therefore moves under the influence of a force whose magnitude is constant but whose direction is always at right angles to the velocity of the particle. The orbit of the particles is therefore circle. Since the centripetal acceleration is v^2/r , we have from Newton's second law:

$$F = qvB = \frac{mv^2}{r} \quad (6.1)$$

Where m is the mass of the particle and r is the radius of the circular orbit of the particle inside the field, therefore :

$$r = \frac{mv}{qB} \quad (6.2)$$

We notice that the radius of the circular orbit directly proportional with (mV) and inversely with the magnetic field (B) .

The angular speed (angular frequency) (ω) of the particle can be obtained as :

$$\omega = 2\pi f = \frac{v}{r} = \frac{qB}{m} \quad (6.3)$$

Similarly the period τ (time required for one complete revolution) is given by :

$$\tau = \frac{2\pi}{\omega} = \frac{2\pi r}{V} = \frac{2\pi m}{qB} \quad (6.4)$$

If the initial velocity of the charged particle has component parallel to the magnetic field B instead of a circle, the resulting trajectory will be a helical as shown in t Fig. (6.1).

Example1 :

Proton inside a magnetic field with intensity 0.2 T, where its velocity perpendicular to the direction of the magnetic field. The proton move in circular orbit with radius 2.0 cm under the effect of the magnetic field.

- Calculate:** 1) The proton velocity in orbit
2) The angular frequency
3) The time required for one complete revolution

Solution :

$$v = \frac{rqB}{m} = \frac{(0.02 \times (m)(1.6 \times 10^{-19} c)(0.2T))}{1.67 \times 10^{-27} kg} = 3.83 \times 10^5 m/s$$

$$\omega = \frac{V}{r} = \frac{3.83 \times 10^5 (m/s)}{0.02(m)} = 1.92 \times 10^7 s^{-1}$$

$$\tau = \frac{2\pi}{\omega} = \frac{2 \times 3.14}{1.92 \times 10^7 s^{-1}} = 3.27 \times 10^{-7} s = 0.237 \mu s$$

6.2 Application on the motion of charge particles in a uniform magnetic field:

6.2.1 The cyclotron:

The cyclotron first made operational 1932 by Ernest O. Lawrence at the university of California, accelerates charged particles such as hydrogen nuclei

(protons and heavy hydrogen nuclei (deuterons) to high energies so that they can be used in atom smashing experiments. We discuss it for two reasons: a) it provides an excellent framework within which to discuss the action of magnetic and electric field on charged particles and b) the cyclotron has led to the production of several generations of improved accelerators.

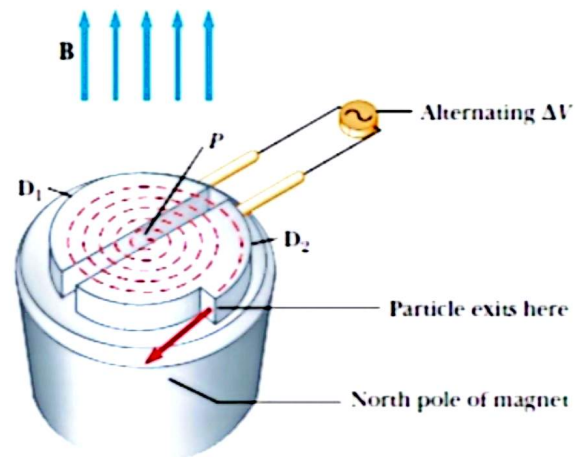
The cyclotron is a machine for speeding up. Sub – atomic particles which exposes to strong magnetic field and compels them to travel in a circular path. The cyclotron proper through a small hole in the wall of the ion source and are available to be accelerated. The cyclotron uses a modest potential difference for accelerating (about 10^5 V), but it requires the ions to pass through this potential difference number of times. To reach 10Mev with 10^5 V accelerating potential requires 100 passages.

A magnetic field is used to bend the ions around so that they may pass again and again through the same accelerating potential. Fig. (6.2) is a top view of the part of the cyclotron that is inside the vacuum. The two D-shaped objects called dees and from part of an electric oscillator which establishes an accelerating potential difference across the gap between the dees. The direction of this potential difference is made to change sign some millions times per second. Finally the space in which the ions move is evacuated to prevent ions from collide with air molecules.

Let us now assume that accelerating potential has now changed sign. Thus the ions again faces a negative dee, is further accelerated and again describes a semicircle of larger radius in the dee. This process goes on until the ion reaches the outer edge of one dee where it pulled out of the system by negatively charged deflector plate. The key to the operation of the cyclotron is that the characteristics frequency ν and

which the ion circulates in the field, we indicate to the frequency potential for accelerating the ion called cyclotron frequency which write to :

$$\nu = \frac{\omega}{2\pi} = \frac{qB}{2\pi m} \quad (6.5)$$



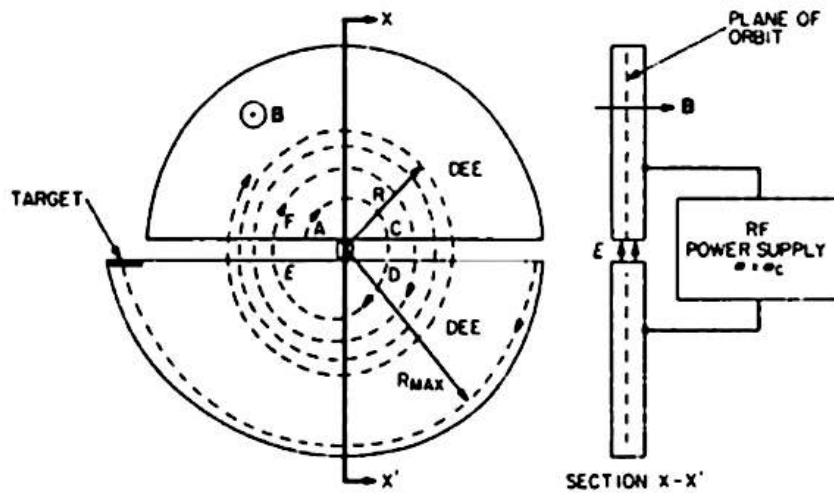


Fig. (6.2)

The energy of the particles produced in the cyclotron depends on the radius (r) of the dees, the velocity of the particles circulating at this radius. The kinetic energy of the particle is then :

$$K.E = \frac{1}{2}mv^2 = \frac{q^2 B^2 r^2}{2m} \quad (6.6)$$

Example:

A small cyclotron had a larger radius of it 2m, used for accelerate protons with intensity magnetic field is 3T. calculate 1) the frequency of the accelerating potential. 2) the kinetic energy of protons when exist from cyclotron.

Solution:

$$v = \frac{qB}{2\pi m} = \frac{(1.6 \times 10^{-19} C)(3T)}{(2 \times 3.14)(1.67 \times 10^{-27} kg)} = 4.58 \times 10^7 Hz$$

$$K.E = \frac{q^2 B^2 r^2}{2m} = \frac{(1.6 \times 10^{-19} C)^2 (3T)^2 (2m)^2}{2(1.67 \times 10^{-27} kg)} = 2.76 \times 10^{-10} J$$

We measure the energy of protons usually with unit MeV where:

$$1MeV = 1 \times 10^6 \times 1.6 \times 10^{-19} = 1.6 \times 10^{-13} J$$

$$K.E = 2.76 \times 10^{-10} \left(\frac{1}{1.6 \times 10^{-13}} \right) = 1.73 \times 10^{-13} \text{ MeV} = 1.73 GeV$$

Example:

A cyclotron had an oscillator frequency of 12×10^6 Hz and a dee radius of 53 cm. What value of the kinetic energy is needed to accelerate deuterons?

Solution:

$$v = \frac{qB}{2\pi m} \Rightarrow B = \frac{2\pi m v}{q}$$

$$B = \frac{(2 \times 3.14)(12 \times 10^6 \text{ Hz})(3.3 \times 10^{-27} \text{ kg})}{1.6 \times 10^{-19} C} = 1.6T$$

$$K.E. = \frac{q^2 B^2 r^2}{2m} = \frac{(1.6 \times 10^{-19} C)^2 (1.6T)^2 (0.53T)^2}{2(3.3 \times 10^{-27} \text{ kg})} = 2.8 \times 10^{-12} J = 17MeV$$

6.2.2 The velocity selector:

In the presence of both electric field E and magnetic field B, the total force on a charge particle is:

$$F = q(E + v \times B) \quad (6.7)$$

This is known as the Lorentz force. By combining the two fields, particles which move with a certain velocity can be selected. This was the principle used by J.J. Thomson as shown in Fig.(6.3). If the charged particles further pass through a region where there exist a down ward uniform electric field, with force qE .

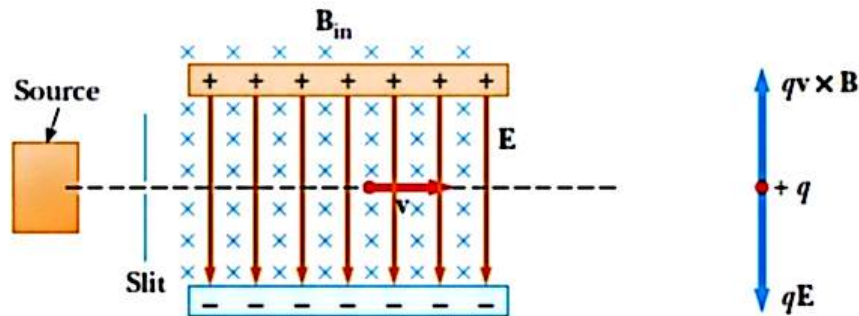


Fig. (6.3)

However, if in addition to the electric field, a magnetic field directed into the page is also applied. Then the particles will experience an additional magnetic force qvB . When the two forces exactly cancel, the particles will move in a straight path. From Lorentz equation we see that when the condition for the cancellation of the two forces is given by :

$$qE = qvB \quad (6.8)$$

which implies. In other words, only those particles with speed $v = E/B$ will be able to move in a straight line. Therefore, the particles with larger velocity deflect to the upper side and particles with lower velocity deflect to down. select a group or beam with the same equal velocity from other particles.

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PROBLEMS

Q1) Protons moves in magnetic field with velocity 3×10^6 m/s, the field gain the proton acceleration 2.4×10^{14} m/s². What is the magnitude of magnetic field.

Ans: 0.8 T

Q2) A charge of particle is 10^{-6} C moves in $+v_e y$ – direction with velocity 3×10^4 m/sec in region of magnetic field in $+v_e x$ – direction, its intensity is 1.5 T, calculate.

i- the force magnitude and its direction of the particle.

ii- If the motion of the particle in $-v_e y$ direction what change in your answer.

Ans : $I = 0.045$ N

II: $=1.045$ N in the opposite direction

Q3) Ionic hydrogen ion move with velocity 3200 m/s near the earth surface, if the intensity magnetic field in the region of moving particle is 1G, compute:

i- the larger magnetic force on the proton.

Ans: $m \times 3200 \times 10^9$ N

Q4) The velocity of proton is 6×10^7 m/s in intensity magnetic field is 2T with force 5×10^{-12} N. What is the angle between the direction of each field and velocity proton.

Ans: $\cos \theta = 0.26$

Q6) Compute the cyclotron frequency of proton inside a magnetic field 3.7T.

Ans: $\omega = 3.7 \times 10^8$ Hz

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Q7) The radius of small cyclotron is 0.5 m, accelerate protons in magnetic field 1.8 T:

- i- What is the potential frequency used.
- ii- What is the kinetic energy of protons when leave cyclotron.

Q8) If used the cyclotron in previous problem to accelerate α -particle.

- i- What is the potential frequency used between dees.
- ii- What is the maximum energy of α - particles.

Q9) The potential difference between two dees in cyclotron equal 60KV. How many periods the proton rotates in cyclotron to become the energy 20 MeV.

Q10) In velocity selector the magnetic field is 0.1 T and electric field is 4×10^5 N/C.

- i- what is the velocity particles passing without deflected.
- ii- what is the kinetic energy of selector protons.
- iii- what is the kinetic energy of selector α -particles.

Q11) Used mass spectrometer to know the isotopes masses of germanium element. If the radius of the isotopes orbitals are 21, 21.6, 21.9 and 22.2cm, if the larger orbital for isotope with mass 76u. (atomic mass). What are the masses of other isotopes.